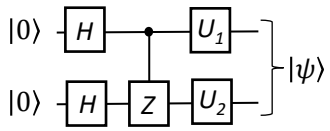
COMPUTER SCIENCE TRIPOS Part II – 2022 – Paper 9

11 Quantum Computing (sjh227)

(a) Consider the circuit:



(i) What should U_1 and U_2 be to prepare the entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$? [2 marks]

(ii) Is state $|\psi\rangle$ always entangled, or does its entanglement depend on U_1 and U_2? Give reasons for your answer. [1 mark]

(iii) If U_1 is a Pauli-X gate and U_2 is a Pauli-Z gate, and the first qubit is sent to 'Alice', and the second to 'Bob', then how can Alice use the shared entangled state to send two (classical) bits of information to Bob, by transmitting only a single qubit? [4 marks]

(iv) If Alice knows only that U_1 is either a Pauli-X or Pauli-Z gate, but Bob knows which of these it is (and U_2 is still a Pauli-Z gate which is known to both Alice and Bob), can Alice still send two bits by transmitting only a single qubit? Explain your answer. [5 marks]

(b) Consider the three-qubit state $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$.

(i) If Alice holds one of the three qubits, and Bob holds the other two, can Alice use this state to teleport one qubit to Bob? Explain your answer. [3 marks]

(ii) If instead Alice and Bob each hold one qubit, and now some third party 'Charlie' holds the third qubit, then give a protocol to enable teleportation between Alice and Bob whereby Charlie first applies a Hadamard gate to his qubit, then measures in the computational basis and sends the measurement outcome to Alice and Bob. [4 marks]

(iii) If Alice, Bob and Charlie each hold one of the three qubits, and there is no communication between Charlie and the other two parties, but Alice and Bob can communicate freely, is there a protocol for Alice and Bob to determine whether or not Charlie has measured his qubit? [1 mark]