COMPUTER SCIENCE TRIPOS Part II – 2022 – Paper 8

11 Quantum Computing (sjh227)

(*a*) A Toffoli gate is to be used as the oracle in the Deutsch-Jozsa algorithm.

(*i*) Why is this not a valid oracle for the Deutsch-Jozsa algorithm? [1 mark]

(*ii*) If the Deutsch-Jozsa algorithm is run anyway with a Toffoli gate as the oracle, what will the outcome be? [6 marks]

(*iii*) How can two Toffoli gates be used to construct an oracle that *is* valid for the Deutsch-Jozsa algorithm? [2 marks]

(*b*) Give a (single qubit) quantum circuit that can perfectly distinguish the states $|+\rangle$ and $|-\rangle$ using any unitary operations, but only computational basis measurements. [2 marks]

(*c*) Show that the quantum states

$$\frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) \quad \text{and} \quad \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

can be perfectly distinguished. Give the measurement basis to achieve this in terms of the computational basis states $|0\rangle$ and $|1\rangle$. [3 marks]

(*d*) Let $|\psi\rangle$ be some unknown quantum state, which is either $|1\rangle$ or $\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$. Furthermore it is known that there is a 75% probability that $|\psi\rangle$ is $|1\rangle$ and a 25% probability that $|\psi\rangle$ is $\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$.

A measurement must be performed to help identify which state $|\psi\rangle$ is. Give a measurement basis that guarantees to correctly determine $|\psi\rangle$ for one of the measurement outcomes; if there are multiple such bases, give the one that maximises the overall probability of correctly identifying $|\psi\rangle$. Give the probability of success. [6 marks]