COMPUTER SCIENCE TRIPOS Part IB, Part II 50% – 2022 – Paper 7

7 Further Graphics (aco41)

(*a*) Recall that the (local) rendering equation is $L_o(\mathbf{x}, \vec{\omega}_o) = L_e(\mathbf{x}, \vec{\omega}_o) + \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i d\vec{\omega}_i$. Assume we can importance sample according to a single function. For each of the following cases, explain what is the best function from the rendering equation to use for importance sampling of the incoming light directions.

(*i*) Diffuse reflection, single light source with a small area, no occluders. [1 mark]

(*ii*) Diffuse reflection, incoming light is the same for all directions, occluders with complex geometry. [2 marks]

(*iii*) Diffuse reflection, incoming light is the same for all direction, no occluders. [1 mark]

(*b*) We are given a scene with a single plane and light sources. The plane does not emit light, has the same BRDF at all points, and light is reflected equally for all outgoing directions.

(*i*) Why can we use the local rendering equation even if we assume global illumination? [2 marks]

(*ii*) Write the simplest expression for the outgoing light at a point on the plane. [2 marks]

(*iii*) Write the expression that approximates the integral with n direction samples that are importance sampled according to the cosine term. You may ignore constants. [2 marks]

(*iv*) We know the incoming light radiance at each geometry point for each direction. We can take n outgoing radiance measurements on the plane. We use the same set of n direction samples at each geometry point to approximate the integral with importance sampling as above (ignoring constants). Compute a discrete approximation of the BRDF sampled at the same direction samples. [*Hint:* For $\mathbf{Ax} = \mathbf{b}$, assume $\mathbf{A}^{-1}$ is known.] [5 marks]

(*c*) We are given a scene where we know that the surface of the only object in the scene has positive Gaussian and mean curvature everywhere and that the surface is diffuse and does not emit light.

(*i*) Derive lower bounds for the minimum and maximum curvatures of the surface at each point. [2 marks]

(*ii*) Assuming smooth BRDF and lighting functions, can we use differentiable rendering directly to estimate scene properties? Briefly explain. [3 marks]