9 Semantics of Programming Languages (nk480)

Consider the following small expression language:

$$
\begin{array}{lrcl}
\text{Expressions} & e & ::= & x \mid \mathsf{let}\, x = e\, \mathsf{in}\, e' \mid \underline{n} \mid e + e' \\
 & & \mid & \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}\, e\, \mathsf{then}\, e'\, \mathsf{else}\, e'' \\
\text{Values} & v & ::= & \underline{n} \mid \mathsf{true} \mid \mathsf{false} \\
\text{Evaluation Contexts} & E & ::= & \mathsf{let}\, x = \Box\, \mathsf{in}\, e' \mid \Box + e \mid v + \Box \\
 & & \mid & \mathsf{if}\, \Box\, \mathsf{then}\, e'\, \mathsf{else}\, e'' \\
\text{Environments} & \rho & ::= & \cdot \mid \rho, x = v
\end{array}
$$

with the following reduction semantics:

$$\frac{\langle \rho; e\rangle \mapsto \langle \rho'; e'\rangle}{\langle \rho; E[e]\rangle \mapsto \langle \rho'; E[e']\rangle} \qquad \frac{x \notin \mathrm{dom}(\rho_1)}{\langle \rho_0, x = v, \rho_1; x\rangle \mapsto \langle \rho_0, x = v, \rho_1; v\rangle}$$

$$\frac{}{\left\langle \rho; \underline{n_1} + \underline{n_2}\right\rangle \mapsto \left\langle \rho; \underline{n_1 + n_2}\right\rangle} \qquad \frac{}{\langle \rho; \mathsf{let}\, x = v\, \mathsf{in}\, e\rangle \mapsto \langle \rho, x = v; e\rangle}$$

$$\frac{}{\langle \rho; \mathsf{if\ true\ then}\, e'\, \mathsf{else}\, e''\rangle \mapsto \langle \rho; e'\rangle} \qquad \frac{}{\langle \rho; \mathsf{if\ false\ then}\, e'\, \mathsf{else}\, e''\rangle \mapsto \langle \rho; e''\rangle}$$

(*a*) Give a set of typing rules for this language. [4 marks]

(*b*) Find an example expression which exhibits an anomalous execution behaviour, and briefly explain what the problem with the semantics is. [5 marks]

(*c*) Modify the semantics to repair the problem, and briefly explain how your solution works. [5 marks]

(*d*) Formulate progress and type preservation lemmas for the repaired language. (A proof is not necessary.) [6 marks]