1 Advanced Algorithms (tms41)

(*a*) Assume you have a randomised approximation algorithm for a maximisation problem, and your algorithm achieves an approximation ratio of 2. What can you deduce for

$$\mathbf{E}[C^*/C],$$

where C^* is the cost of the optimal solution, C is the cost of the solution of the approximation algorithm, and $\mathbf{E}[.]$ denotes the expectation? [4 marks]

(*b*) Consider the following optimisation problem on graphs: Given an undirected, edge-weighted graph $G = (V, E, w)$ with $w : E \to \mathbb{R}^+$, we want to find a subset $S \subseteq V$ such that $w(S, V \setminus S) = \sum_{e \in E(S, V \setminus S)} w(e)$ (the total sum of weights over all edges between S and $V \setminus S$) is maximised.

(*i*) Design a polynomial-time approximation algorithm for this problem. Also analyse its running time and prove an upper bound on the approximation ratio. [8 marks]

(*ii*) Find a graph which matches your upper bound on the approximation ratio from Part (*b*)(*i*) as closely as possible. [4 marks]

(*iii*) Consider now the following generalisation of the problem. Given an integer $k \geq 2$, we want to partition V into disjoint subsets $S_1, S_2, \ldots, S_k$ so that we maximise

$$\sum_{i=1}^{k} w(S_i, V \setminus S_i).$$

Describe an extension of your algorithm in Part (*b*)(*i*). What approximation ratio can you prove for this algorithm? [4 marks]