1 Advanced Algorithms (TMS)

(*a*) What are the three possible cases for the solution of a linear program? For each of them, give an example of a linear program in standard form exhibiting this case. [6 marks]

(*b*) What is the set of optimal solutions for the following linear program?

$$\begin{aligned} \text{Minimize} \quad & -x_1 - x_2 \\ & -x_2 \geq -3 \\ & 2x_1 + x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

[6 marks]

(*c*) For a given linear program $\mathbf{LP}_1$

$$\begin{aligned} \text{Maximize} \quad & \sum_{j=1}^{n} c_j x_j \\ & \sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad (1 \leq i \leq m) \\ & x_j \geq 0 \qquad (1 \leq j \leq n), \end{aligned}$$

consider a new linear program $\mathbf{LP}_2$:

$$\begin{aligned} \text{Minimize} \quad & \sum_{i=1}^{m} b_i y_i \\ & \sum_{i=1}^{m} a_{ij} y_i \geq c_j \qquad (1 \leq j \leq n) \\ & y_i \geq 0 \qquad (1 \leq i \leq m). \end{aligned}$$

(*i*) Prove that if x is a feasible solution for $\mathbf{LP}_1$ and y is a feasible solution for $\mathbf{LP}_2$, then $c^T x \leq b^T y$. [6 marks]

(*ii*) Using your answer in Part (*c*)(*i*), what can we conclude about $\mathbf{LP}_2$ if we know that $\mathbf{LP}_1$ is unbounded? [2 marks]