COMPUTER SCIENCE TRIPOS Part II – 2017 – Paper 9

6 Digital Signal Processing (MGK)

(*a*) A discrete-time LTI filter can be described through the locations of zeros and poles in the z-transform $H(z)$ of its impulse response. Consider an IIR filter of order 2 with $H(c_1) = H(c_2) = 0$ and $|H(z)| \to \infty$ for $z \to d_1$ and $z \to d_2$.

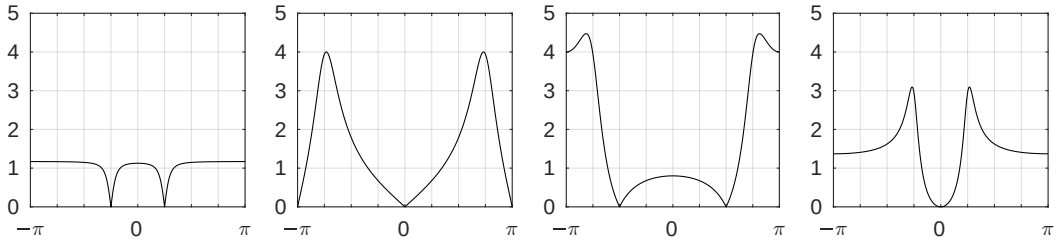
(*i*) What is the z-transform $H(z)$ of its impulse response? [2 marks]

(*ii*) What additional parameter (beyond c_1, c_2, d_1, d_2) is required to fully describe the impulse response of this filter? [1 mark]

(*iii*) What is the magnitude of the discrete-time Fourier transform (DTFT) of the impulse response of this filter? [2 marks]

(*iv*) Under what condition on c_1, c_2, d_1, and d_2 is the impulse-response of this filter real-valued? [2 marks]

(*b*) The following plots show the magnitude of the DTFT of the real-valued impulse response of four different IIR filters:



(*i*) (*ii*) (*iii*) (*iv*)

The z-transform of each impulse response has two zeros and two poles. Each zero or pole is at one of these 12 possible locations: $\mathrm{e}^{\pi \mathrm{j} k/4}$ with $k \in \{0, \ldots, 7\}$ or $0.6 \pm 0.6\mathrm{j}$ or $-0.5 \pm 0.5\mathrm{j}$.

For each filter, state the location of both zeros and both poles. Explain the reasoning behind your choice. [8 marks]

(*c*) What is the z-transform $H(z)$ of the impulse response of the following filter? [5 marks]