COMPUTER SCIENCE TRIPOS Part II – 2016 – Paper 9

1 Advanced Algorithms (TMS)

(a) Explain the difference between PTAS and FPTAS, and give one example of a problem for which a FPTAS is known, and one example of a problem for which a PTAS is known but no FPTAS. [4 marks]

(b) We consider an extension of the MAX-3-CNF problem, called MAX-4-CNF problem, where we are given a 4-CNF formula with m clauses, e.g., $(x_1 \vee \overline{x_3} \vee x_4 \vee x_5) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3 \vee \overline{x_5}) \wedge \cdots$, and the goal is to find an assignment of the variables $x_1, x_2, \ldots, x_n$ that satisfies as many clauses as possible.

(i) Design a randomised approximation algorithm and analyse its approximation ratio. (For full marks, the approximation ratio must be smaller than 10/9.) [4 marks]

(ii) Express the MAX-4-CNF problem as an integer program. [4 marks]

(iii) Based on the construction from Part (b)(ii) or otherwise, describe an algorithm that performs randomised rounding on the solution of a linear relaxation. [3 marks]

(iv) Analyse the expected approximation ratio of the algorithm from Part (b)(iii).
Hint: You may want to use the following two inequalities. Firstly, for any non-negative numbers $a_1, a_2, \ldots, a_k$, we have

$$\left(\prod_{i=1}^{k} a_i\right)^{1/k} \leq \frac{\sum_{i=1}^{k} a_i}{k}.$$

Secondly, for any integer $k \geq 2$ and $0 \leq a \leq 1$,

$$1 - \left(1 - \frac{a}{k}\right)^k \geq \left(1 - \left(1 - \frac{1}{k}\right)^k\right) \cdot a.$$

[5 marks]