5 Digital Signal Processing (MGK)

A discrete sequence $\{x_n\}$ can be converted into a continuous representation

$$\hat{x}(t) = t_{\mathrm{s}} \cdot \sum_{n=-\infty}^{\infty} \delta(t - n \cdot t_{\mathrm{s}}) \cdot x_n,$$

where t_{s} is the sampling period.

(*a*) State two characteristic properties of Dirac's δ function. [2 marks]

(*b*) Describe briefly how this representation helps to explain aliasing. [4 marks]

(*c*) Define three functions $h(t)$, such that convolving $\hat{x}(t)$ with $h(t)$ results in

(*i*) the output of an idealized analog-to-digital converter that holds the output voltage of each sample x_n for the time interval from $t = n \cdot t_{\mathrm{s}}$ until the next sample x_{n+1} arrives at time $t = (n+1) \cdot t_{\mathrm{s}}$; [4 marks]

(*ii*) linear interpolation of $\{x_n\}$; [4 marks]

(*iii*) reconstruction of a signal $x(t)$ that was sampled as $x_n = x(n \cdot t_{\mathrm{s}})$, assuming that the Fourier transform of $x(t)$ is zero at any frequency f with $|f|^{-1} \leq t_{\mathrm{s}}$ or $|f|^{-1} \geq 2t_{\mathrm{s}}$. [6 marks]