6 Digital Signal Processing (MGK)

BBC Radio Cambridgeshire broadcasts a radio signal on a carrier frequency of 1026 kHz with a (double-sided) bandwidth of 10 kHz. You connect a long wire (antenna) via an amplifier and bandpass filter (0.5–2.0 MHz) to an analog-to-digital converter (ADC) with a sampling frequency of 5 MHz.

(*a*) You record $n = 500$ consecutive samples $(x_0, x_1, \ldots, x_{499})$ from the analog-to-digital converter output and calculate the Discrete Fourier Transform (DFT)

$$X_k = \sum_{i=0}^{n-1} x_i \cdot \mathrm{e}^{-2\pi\mathrm{j}\frac{ik}{n}}$$

(*i*) For which index value(s) k do you expect $|X_k|$ to best indicate the received signal strength of this radio station? [4 marks]

(*ii*) What preprocessing step would improve this indication? [4 marks]

(*iii*) What redundancy do you expect to find in the DFT output vector X, considering that the input signal is real-valued? [2 marks]

(*iv*) Explain a technique that exploits this redundancy to calculate this real-valued DFT more efficiently. (You can assume that an FFT implementation is already available and that $n = 512$ is used instead.) [4 marks]

(*b*) You want to record the output of this radio station for later analysis, but you do not yet know how it was modulated. How can you convert the ADC output sequence $\{x_i\}$ such that the resulting sequence encodes efficiently what is happening in the frequency range 1021–1031 kHz, with as low a sample rate as possible? [4 marks]

(*c*) You finally learn that the signal recorded in (*b*) was an amplitude-modulated positive mono audio signal. How can you demodulate it? [2 marks]