2010 Paper 8 Question 6

Digital Signal Processing

The *Purpletoe* standard for trouser-area networking uses a radio signal with a bandwidth of less than 1 MHz. The carrier frequency is $f_c(k) = (2400 + 2k)$ MHz, where $k \in \{1, 2, 3, \ldots, 40\}$ is the channel number. Consider a receiver design in which the antenna signal is first multiplied with a sine wave of *fixed* frequency f_m, is then band-pass filtered to eliminate frequencies outside the range 1 MHz to 100 MHz, and is finally sampled by an analogue-to-digital converter with sampling frequency f_s for further digital processing.

(*a*) What is the largest set of frequencies from which f_m can be chosen such that no information is lost from any of the 40 channels? [4 marks]

(*b*) Which of the combinations of f_m and f_s that preserve all information from all 40 channels in the sampled output has the lowest sampling frequency f_s, assuming there is no signal outside these channels? [4 marks]

(*c*) To make eavesdropping more difficult, *Purpletoe* transmitters hop several times each second from one channel to another, in a secret pseudo-random order that is cryptographically pre-agreed and shared only with intended receivers. Consider for your receiver a special eavesdropping mode that exploits aliasing such that transmissions of a data packet using different channel numbers k all look the same after sampling (assuming that there is only a single transmitter in range). Which combination of f_s and f_m achieves that, and how? [8 marks]

(*d*) Cost pressures force you to use a cheaper circuit that multiplies the radio signal with a square wave of frequency f_m, instead of a sine wave. How does this affect the design of your receiver? [4 marks]