2006 Paper 9 Question 10

Digital Signal Processing

Consider a software routine that converts the sampling rate of digital audio data from 8 kHz to 48 kHz, without changing the represented sound. It reads an input sequence $\{x_i\}$ and produces an output sequence $\{y_i\}$. The routine first inserts five samples of value 0 between each consecutive pair of input samples. This results in a new intermediate sequence $\{x'_i\}$ with $x'_{6i} = x_i$ and $x'_{6i+k} = 0$ for all $k \in \{1, \ldots, 5\}$. The sequence $\{x'_i\}$ is then low-pass filtered, resulting in $\{y_i\}$.

(*a*) How can the process of taking discrete-time samples $\{x_i\}$ from a continuous waveform $x(t)$ be modelled through a function $\hat{x}(t)$ that represents the sampling result but can still be analysed using the continuous Fourier transform? [2 marks]

(*b*) What effect does sampling with 8 kHz have on the Fourier spectrum of the signal? [2 marks]

(*c*) How and under what condition can this sampling process be reversed? [2 marks]

(*d*) Can $\hat{x}(t)$ also model another sampling process that results in the discrete sequence $\{x'_i\}$, and if so, what is its sampling frequency? [2 marks]

(*e*) How does the continuous spectrum associated with $\{x'_i\}$ relate to that of $\{x_i\}$? [2 marks]

(*f*) What purpose is served by the low-pass filter that the routine applies? In particular, what would happen to a 1 kHz sine tone input if this filter were not applied and $\{y_i\} = \{x'_i\}$ were output instead? What cut-off frequency must the filter have? [5 marks]

(*g*) Provide a formula for calculating a 25-sample long causal finite impulse response $\{h_i\}$ of a low-pass filter suitable for this routine, based on the Hamming windowing function. [5 marks]