2005 Paper 7 Question 10

Digital Signal Processing

(a) Characterise the systems below as linear/non-linear, causal/non-causal, and time invariant/time varying:

 (i) $y_n = a x_{3n-2}$ [2 marks]

 (ii) $y_n = y_{n-1} + 6 x_{n-2}$ [2 marks]

 (iii) $y_n = y_{n-1} - x_{n+5} + x_{n-5}$ [2 marks]

 (iv) $y_n = \frac{x_n}{x_{n-3} y_{n-2}}$ [2 marks]

 (v) $y_n = x_n - \cos\left(\frac{\pi}{2} n\right)$ [2 marks]

(b) Consider the system $h : \{x_n\} \to \{y_n\}$ with $y_n - y_{n-1} = x_n - x_{n-3}$.

 (i) Give the impulse response of this system. [2 marks]

 (ii) Give one sine-wave input sequence of the form

$$x_n = a \cdot \sin(b \cdot n + c)$$

 (with $a \neq 0, b \neq 0$) for which $y_n = 0$ for all n. [2 marks]

 (iii) Express the system h as a rational function $H(z)$. [3 marks]

 (iv) Determine the values $z \in \mathbb{C}$ for which $H(z) = 0$. [3 marks]