2002 Paper 8 Question 8

Advanced Algorithms

(a) Explain what is meant by the Kolmogorov Complexity $K(n)$ of a natural number n. [5 marks]

(b) Consider a graph of the function $K(n)$ plotted against n:

(i) Show that it is smooth, in the sense that for any n and fairly small value of k the value of $K(n+k)$ will be quite close to the value of $K(n)$. [3 marks]

(ii) Show that it is rough, in the sense that for any N there are two values of n_1 and n_2 between N and $2N$ such that $K(n_1)$ is about $2^{K(n_2)}$, i.e. one has a complexity exponentially bigger than the other. [3 marks]

(iii) Explain why the graph is bounded above by some straight line of the form $n+c$ and comment on what the constant represents. [3 marks]

(iv) Explain why for any constant k there will be a value N such that $n > N$ implies that $K(n) > k$. [3 marks]

(v) Demonstrate that there is no constant N such that $n > N$ implies $K(n) > \log\log\log\log n$. [3 marks]